Summer 2015

Designing a Self-reversing Track Layout with TrackMaster Tracks

Breanna M. Struss
Valparaiso University, breanna.struss@valpo.edu

Seth Hamilton
Valparaiso University, seth.hamilton@valpo.edu

Designing a Self-reversing Track Layout With TrackMaster™ Tracks

Seth Hamilton and Breanna Struss

Advisor: Dr. James Caristi

Abstract

This research project sought to find general track formations that allowed battery operated locomotives to traverse the entire train track in both directions infinitely. These formations allowed for any number of track pieces from TrackMaster™ Thomas and Friends™ sets by Fisher Price®. The research looked at start position of the train and presetting of the switches as well as what is necessary to have a complete track with no dead ends. Surprisingly, there was found to be only one track formation that allowed for entire traversal of the track in both directions. This layout was termed a "dog bone" and consisted of two switch pieces connected at the ends with the tips of both switches connecting to themselves on the same switch. A proof that this layout is the only layout that satisfies the conditions is given.

Foundations

Here are some simple definitions that will build a foundation for the latter portions of the article.

- **Definition 1.1** An *end* is an entry point to the switch piece such that when the train enters the end, it can travel in one of two directions but cannot change the switch's state (part c in **Figure 5**). A *tip* is an entry point to the switch piece such that a train entering the switch from a tip must exit through the end, and the train could change the switch's state (parts a and b in **Figure 5**).
- **Definition 1.2** A *self-reversing track* is a track layout in which a train infinitely traverses part of the track in opposite directions.
- **Definition 1.3** A *dog bone* is a track layout in which two switch pieces are connected to each other at the ends, and the tips of the same switch are connected. (see **Figure 3**) It can be seen that the dog bone may be extended and made more "complicated."
- **Definition 1.4** An *arc* is any connection between a tip and another tip of the same or different switch piece. (*note a tip to an end is not an arc and an end to an end is not an arc)
- The initial conditions of the track layout such as the position of the train and switches, as well as the direction of the train have no ability to make a layout that doesn't work a layout that works.

Train

Switch

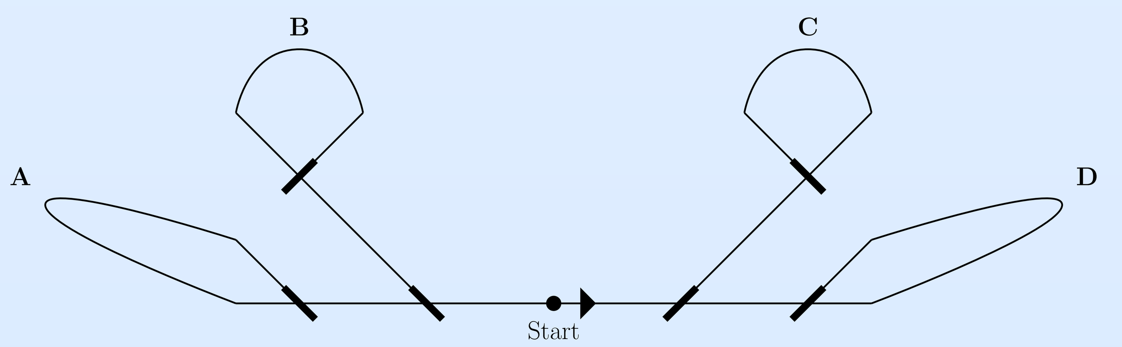

Figure 1

- In this figure, depending on the initial conditions, a train will traverse one dog bone out of 4 possibilities. The train can traverse a dog bone between arcs **A** and **D**, **A** and **C**, **B** and **D**, or **B** and **C**.

- **Definition 1.5** *Dead track* is track that is not infinitely traversed.
- **Definition 1.6** A *closed track* is a track layout such that for every piece of track, every entry point must be connected to another entry point. (*note: A track that is not closed will have dead track)
- **Definition 1.7** A switch is in position *0* if it is all the way left as viewed from the end of the switch. A switch is in position *1* if it is all the way right as viewed from the end of the switch.

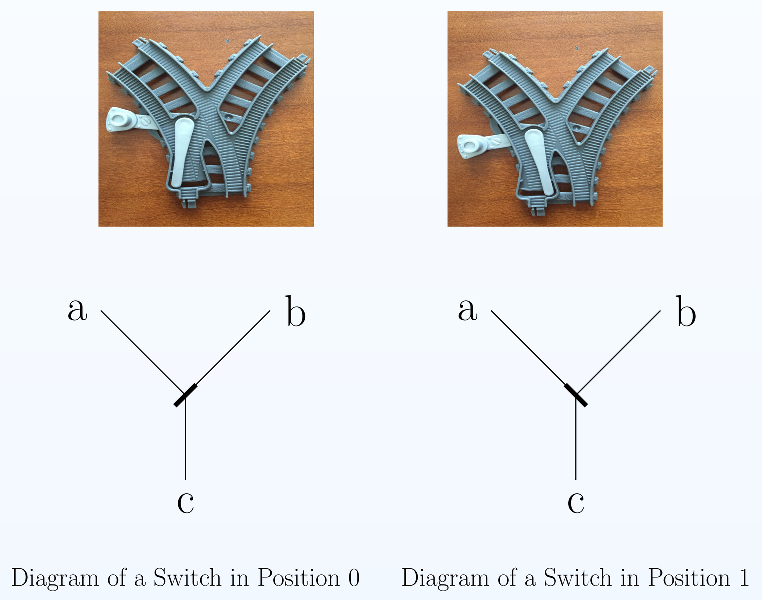

Figure 2

Why the dog bone is the only layout that works

1st Traversal

2nd Traversal

Switch

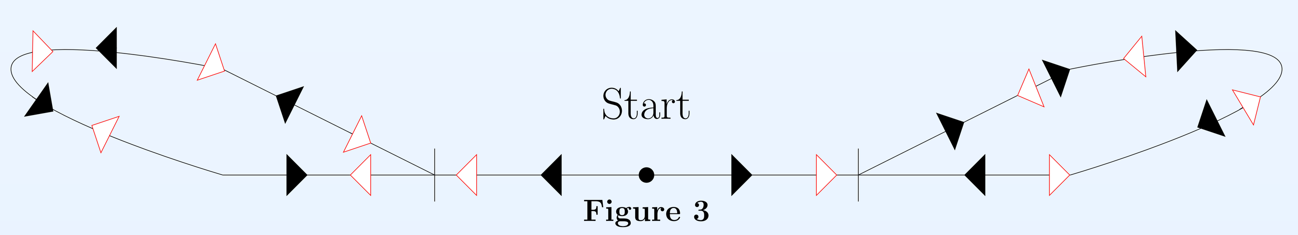

Figure 3

Theorem *Any track layout that is self-reversing with no dead track is a dog-bone.*

Lemma: *If a train enters a finite set of tracks, S, at an entry point, x and exits S at an entry point, y, and proceeds directly to x without entering any part of S, then the train will continually exit S at y.*

Proof: of Lemma

Consider **Figure 4**. Assume a train enters a set of track, S, at x and exits S at y one time. Let P be the first switch piece in S where x is connected to P. During the first traversal, if P connects to x at an end, then the train cannot re-enter P without violating the assumption of the **Lemma**. If P connects to x at a tip, then the train must exit P through the end. Thus, the train will traverse the same path through the switch piece P in both the first and the second traversal, because the switch's state will not be changed. Now, consider **Figure 5**. Let x' be the exit point of the switch P that is used in both the first and the second traversals. Let S' be the set of all track in S connected to x' and y. This is the same as **Figure 4** with one less switch piece. We now have a situation in which x' acts as x and the train still exits S' at y. This argument can be applied recursively for every switch piece in S until there are no switch pieces left, and thus the train will infinitely traverse from x to y the same way each time. □

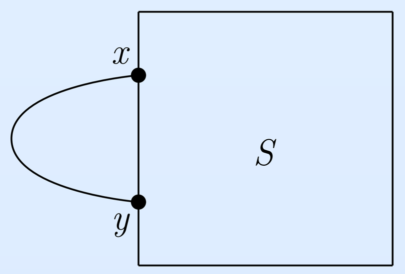

Figure 4

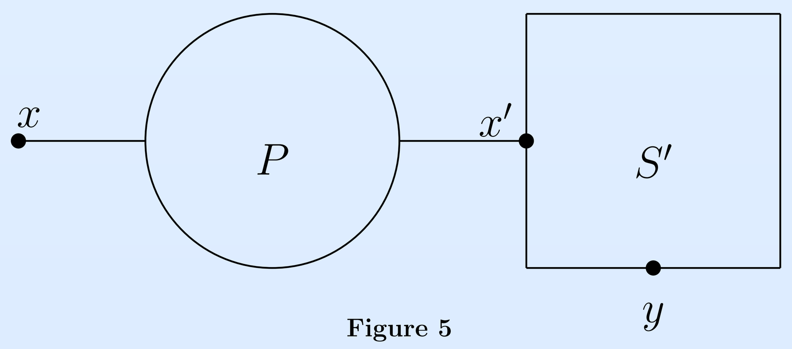

Figure 5

Corollary *In any track layout with no dead track, the end of a switch must connect to the end of another switch.*

If a tip is connected to an end, then when the train exits at the tip, it will pass through the end and re-exit the switch at the tip, in which case the **Lemma** applies. Similarly, if the train exits at an end and re-enters the tip, it is forced to exit through the end, and the **Lemma** applies again. In both cases there is dead track due to the other tip that is never traversed. This violates the assumption. Therefore, an end can only be connected to another end.

Proof of the Theorem

Proof. Assume we have a self-reversing track layout with no dead track.

By the **Corollary**, we know that a track layout with no dead track must have the ends of every switch piece connected to another end of a different switch piece. Consider **Figure 6** where $a, b, c,$ and d are tips and R and T are switch pieces. If the train traverses from a tip of R to a tip of T, the train will infinitely traverse the track in a continuous loop because the switches cannot change their state and the **Lemma** applies. This results in dead track. Consider **Figure 7**. If the train exits at the tip, a, of R and re-enters at a, then tip b must also connect to itself, because we know that a tip from R cannot connect to a tip on T. So, when both tips connect to themselves, there is dead track since either a or b will not be reachable due to the train's inability to change the state of the switch. Therefore, the only possible connection is for the tips of R to arc to each other and for the tips on T to do the same. Thus, we have two switches that connect to each other at the ends and have self-arcing tips. Therefore, any track layout that allows continual traversal of the track in both directions, with no dead track is in fact the dog bone layout as described in **Definition 1.3**, with two switches connected at their ends and self-arcing tips on both switch pieces. □

Switch

Figure 6

Switch

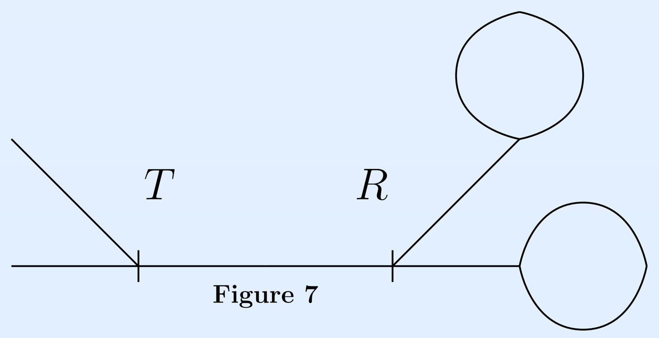

Figure 7

If you want to know more about this topic, please email us at **breanna.struss@valpo.edu** and **seth.hamilton@valpo.edu**.